**LU Decomposition:**

***Step 1(****LU decomposition step****):***

Coefficient matrix is factored or “decomposed” into lower and upper triangular matrices.

***Step 2(****Substitution step****):***

and are used to determine a solution for a right-hand side .

This step itself consists of two steps.

1. First an intermediate vector is generated by forward substitution.
2. Then, the result is substituted, to get the, which can be solved by back substitution.

**Methods:**

1. **Doolittle** decomposition, provided that: all of diagonal elements is 1 and diagonal elements is not zero.
2. **Crout** decomposition, provided that: diagonal elements is not zero and all of diagonal elements is 1.
3. **Cholesky** decomposition, provided that: is symmetric matrix. And

with and diagonal elements is not zero.

**Doolittle Method.**

A system of linear equations can be written as

And

Where

,

Substituting the matrix

We have,

\*

So, the values of upper and lower triangular matrix can be calculated as.

, ,

**Question:**

The following system of equations is designed to determine concentrations (the c’s in ) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day)

1. Determine the solution.
2. Determine how much the rate of mass input to reactor 3 must be increased to induce a 10 rise in the concentration of reactor 1.

**Solution: (a)**

A system of linear equations can be written as

Where

,

**LU decomposition:**

To find the values of matrix and

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | |
|  | | |

**Substitution:**

Forward substitution

Backward substitution

**(b).**

g/day of mass input to reactor 3 must be decreased to induce a 10 rise in the concentration of reactor 1.

**Crout Method.**

**A system of linear equations can be written as**

And

Where

,

Substituting the matrix

We have,

\*

So, the values of upper and lower triangular matrix can be calculated as.

,

,

,

**Question:**

A civil engineer involved in construction requires 4800, 5810, and 5690 of sand, fine gravel, and coarse gravel, respectively, for a building project. There are three pits from which these materials can be obtained. The composition of these pits is

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Sand**  **%** | **Fine gravel**  **%** | **Coarse gravel**  **%** |
| **Pit 1** | 52 | 30 | 18 |
| **Pit 2** | 20 | 50 | 30 |
| **Pit 3** | 25 | 20 | 55 |

How many cubic meters must be hauled from each pit in order to meet the engineer’s needs?

**Solution:**

A system of linear equations can be written as

And

Where

,

**LU decomposition:**

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | 0.3846 | |
|  |  | |
| 0.23077 |  | |

**Substitution:**

Forward substitution

Backward substitution

**Answer:**

cubic meters must be hauled from pit 1, pit 2 and pit 3 respectively, in order to meet the engineer’s needs.

**Cholesky Method.**

A symmetric matrix is one where for all and. In other words,

=. Such systems occur commonly in both mathematical and engineering problem contexts. They offer computational advantages because only half the storage is needed and, in most cases, only half the computation time is required for their solution.  
One of the most popular approaches involves Cholesky decomposition. This algorithm is based on the fact that a symmetric matrix can be decomposed, as in

That is, the resulting triangular factors are the transpose of each other.

A system of linear equations can be written as

Where is symmetric matrix, and

Where

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

**Question:**

Find the solutions by using Cholesky decomposition.

**Solution:**

**LU decomposition:**

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

**Substitution:**

Forward substitution

Backward substitution

**Answer:**

The solution of system of linear equations is,